1) **133** **Work Backwards.** *This is a classic WB (Work Backwards) problem: we are given the final result (109) but need to find the original number. To do this, you perform the opposite operation and go one step at a time.*

<u>Add 10</u>: Subtract 10. 109 – 10 = 99.
<u>Multiply by 9</u>: Divide by 9. 99 / 9 = 11.
<u>Subtract 8</u>: Add 8. 11 + 8 = 19.
*<u>Divide by 7</u>: Multiply by 7. 19 x 7 = **133**.*

Confirm by checking. 133 / 7 = 19 – 8 = 11 x 9 = 99 + 10 = 109: YES.

2) **160** **Area of Squares; Perimeter.** *Begin by COUNTING the number of squares: there are 28. Next, divide 700 by 28 to get the area of each square. 700 / 28 = 25. With an area of 25 sq meters, the side of each square must be 5, because 5 x 5 = 25.*

Now, COUNT the total number of sides along the H-shrine. Count carefully, because there are a lot of sides!

*There are 32 total sides, each of length 5 meters. Finally, multiply: 32 x 5 = **160**, which is the perimeter of the H-shrine.*

3) **119** **RTQ.** *At first this seems like a strange question, but if you read it carefully, you will realize that it's nothing more than an addition problem! Be sure you understand what to do by reading the example, then apply that understanding to the problem itself.*

<u>Starting value of T</u>: 2.
<u>T to H</u>: 12. <u>H to E</u>: 3.
<u>E to T</u>: 15. <u>T to I</u>: 11. <u>I to N</u>: 5. <u>N to G</u>: 7.
<u>G to G</u>: 0. <u>G to O</u>: 8. <u>O to E</u>: 10. <u>E to S</u>: 14.
<u>S to S</u>: 0. <u>S to K</u>: 8. <u>K to R</u>: 7. <u>R to A</u>: 17. <u>A to A (all remaining combinations)</u>: 0.

*The total value is 2 + 12 + 3 + 15 + 11 + 5 + 7 + 0 + 8 + 10 + 14 + 0 + 8 + 7 + 17 + 0. Whenever you have a long list of numbers to add, be sure to add it in TWO DIFFERENT WAYS to make sure you get the same answer both times. Often, the best way to do this step is to group by 10's, 20's, 30's, etc. We get 10 + 10 + 20 + 30 + 25 + 24 = **119**.*

4) **7** **Logic; G/C.** *This excellent problem, composed by student Sneha Kancharla, can be solved in multiple ways. We'll show two.*

<u>Method 1: G/C</u>. Start with 4's since that's the biggest number.

- *If Bidipta buys **6** 4's, then he has used up all $24 for only 6 pairs of socks. No.*
- *If he buys **5** 4's, he has used 5 x 4 = $20, leaving $4 for 7 more pairs of socks. No.*
- *If he buys **4** 4's, that's $16 spent, leaving $8 for 8 pairs. He could buy all 1's, but then he wouldn't have anything left to spend on 3's. No.*
- *If he buys **3** 4's, that's $12 spent, leaving $12 for 9 pairs. Does this work? 8 1's + 1 3's = $11; no. 7 1's + 2 3's = $13; no. Additional 3's will take us over $13, so this does not work.*
- *If he buys **2** 4's, that's $8 spent, leaving $16 for 10 pairs. Does this work? 9 1's + 1 3's*

= \$12; no. 8 1's + 2 3's = \$14; no. 7 1's + 3 3's = \$16: YES! Bidipta therefore buys 2 pairs of \$4 socks, 3 pairs of \$3 socks, and 7 ***pairs of \$1 socks*** *(8 + 9 + 7 = \$24 total; 2 + 3 + 7 = 12 pairs of socks total).*

<u>Method 2: Average</u> (solution provided by Sneha). The total cost of the socks is \$24, and Bidipta bought 12 pairs of socks. Then the average cost is \$24 ÷ 2 = \$2 per pair.

There are 2 ways to make an average of \$2:

- ***2 \$1 pairs and 1 \$4 pair****. That adds up to \$6 and 3 pairs. \$6 ÷ 3 = \$2 per pair.*
- ***1 \$1 pair and 1 \$3 pair****. That adds up to \$4 and 2 pairs. \$4 ÷ 2 = \$2 per pair.*

Let A represent the number of \$6 packages and B represent the number of \$4 packages. Then we see that 6A + 4B = \$24. Determine A and B using G/C.

- *If A = 4, then B = 0; this is impossible because then Bidipta doesn't buy any \$3 pairs.*
- *If A = 3, then 18 + 4B = 24, which means 4B = 6. This is impossible because B cannot be a mixed number.*
- *If A = 2, then 12 + 4B = 24, which means 4B = 12, so that B = 3. This works!*

Bidipta therefore bought 2 A packages and 3 B packages. That's 4 \$1's + 2 \$4's for the A's and 3 1's + 3 3's for the B's. In total, that is 7 \$1's, 3 \$3's, and 2 \$4's, which makes \$7 + \$9 + \$8 = \$24 total for 7 + 3 + 2 = 12 pairs of socks. ***Bidipta bought 7 pairs of \$1 socks****.*

5) **42** **Logic; CLT.** *We are given a variety of statements, all of which must be true. It makes sense to begin with the statement that restricts us the most, so that we have fewer possibilities to consider.*

<u>Today, Dilly is three years older than Billy</u>: too many possibilities; skip it.

<u>In 7 years, B = 2 x S</u>: again, too many possibilities.

<u>In 2 years, D = 2 x W and D = 4 x S</u>: this is better. D must be a multiple of 2 and 4.

<u>4 years ago, D = 2 x F and D = 5 x W</u>. This is the best one, because D must be a multiple of both 2 and 5, which means he must be a multiple of 10. Start with the simplest case and see if it works: four years ago, D = 10, which means that F = 5 and W = 2.

This means that TODAY, D = 14, F = 9, and W = 6.
In 2 YEARS, D = 16, F = 11, and W = 8.
In 7 YEARS, D = 21, F = 16, and W = 13.

Check to see if each statement works. If D is three years older than Billy today, then D = 14 and B = 11.

If B = 11 today, then in 7 years, B = 18, which means that S = 9 at that time.

In 2 years, we know that D = 16 and W = 8 (which checks out since D = 2 x W in two years). Then since D = 4 x S, S must be 4 at that time.

The list of ages we know at this point looks like this:

	4 yrs. ago	**Today**	**in 2 yrs.**	**in 7 yrs.**
D	*10*	*14*	*16*	*21*

B		*11*		*18*
F	*5*	*9*	*11*	*16*
W	*2*	*6*	*8*	*13*
S			*4*	*9*

Fill in the remaining blanks and then check the original statements to ensure everything works.

	4 yrs. ago	**Today**	**in 2 yrs.**	**in 7 yrs.**
D	*10*	*14*	*16*	*21*
B	*7*	*11*	*13*	*18*
F	*5*	*9*	*11*	*16*
W	*2*	*6*	*8*	*13*
S	--	*2*	*4*	*9*

*All of the information does check out. We are looking for the total combined ages of the five kids currently. Add up their ages today: 14 + 11 + 9 + 6 + 2 = **42**.*

6) **65292** **Logic (Secret Password), G/C.** *As with all Secret Password questions, begin by setting up the number of spaces (in this case five, since it has five digits) along with your Box O' Digits.*

Begin with the most restrictive statement. The first digit is three times the 5th digit. This means that the 1st digit can be 0, 3, 6, or 9; eliminate 0 because then the password isn't a 5-digit number. The corresponding 5th digit would be 1, 2, or 3.

Use the next most restrictive statement. The fourth digit is 4 more than the 2nd digit. This means that the 4th digit can be anything from 4-9, while the corresponding 2nd digit ranges from 0-5.

Combine the remaining statement with what we know from the statement above. The 3rd digit is 3 less than the 2nd digit. This means that the 3rd digit can be anything from 0 to 6, while the corresponding 2nd digit would be anything from 3 to 9. But we know from the second statement that the 2nd digit can only be from 0-5. The second digit must work with both statements, so it must be 3, 4, or 5. This means that the 4th digit must be 7, 8, or 9.

Summarize the information into your spaces:

3, 6, 9	*3, 4, 5*	*0, 1, 2*	*7, 8, 9*	*1, 2, 3*

At first, this might seem like a huge number of possibilities, but keep in mind that the 1st and 5th digits must go together, while the 2nd, 3rd, and 4th digits must go together as well. There are 3 possible groupings for the 1st and 5th digits, and there are 3 possible groupings for the 2nd, 3rd, and 4th digits. That gives us 3 x 3 = 9 possible answers. List these:

33071
34181
35291
63072
64182

65292
93073
94183
95293

Of these 9 possibilities, only one includes three different pairs of digits that add up to 11. 6 + 5 = 11; 2 + 9 = 11; 9 + 2 = 11. The code to Sude's safe is ***65292****.*

7) **40** **Logic; Process of Annihilation.** *Originally, I solved this problem by focusing on the middle and right columns, listing all possible digits that could go in those boxes. I determined that the only two digits that didn't work for any of those boxes were 5 and 7; therefore, those digits had to go in the left column. I then figured out which numbers went in which box from there.*

That method works, and it did lead to the correct answer. However, there is a more direct method which is faster and easier. ***Nathaniel*** *provides it as follows:*

This problem has a lot of different restrictions. To make it easier from the start, <u>let's begin solving by using the most restrictive fact: that C × V ÷ Z = 27</u>. We can see from this that C × V has to be a multiple of 27, so C and V have to be either 9 and 3 or 9 and 6.

If they are 9 and 3, Z must be 1. However, since X ÷ Y – Z = 0, there is a problem: if Z is 1, then X ÷ Y must also be 1, which means that X = Y. That is not possible, so ***Z must be 2****, and C and V are 9 and 6.*

Now since T ÷ U – V = 1, V must be 6: if it were 9, then T ÷ U = 10, which is again impossible. Therefore, ***V = 6, meaning that C = 9*** *and T ÷ U = 7. It is then clear that* ***T is 7 and U is 1****, as those are the only digits whose quotient is 7.*

Now, notice that X ÷ Y – Z = 0. Since Z = 2, X ÷ Y must also be 2. Since 2 and 6 have already been used, ***X and Y have to be 8 and 4 respectively****.*

Finally, because B ÷ U × Y = 12, ***B is 3*** *(3 × 4 = 12)* ***and A, the last remaining digit, is 5*** *(5 + 3 + 9 = 17). Our final answer, Y × A × Z, is 4 × 5 × 2, or* ***<u>40</u>****.*

The completed chart looks like this:

5	**3**	**9**
7	**1**	**6**
8	**4**	**2**

8) **1024** **G/C or Logic.** *How in the world are you supposed to solve a problem when it seems like it will have an infinite number of possible solutions?!*

Answer: ***start somewhere*** *and see if you notice something. We'll present two solutions.*

*Method 1: G/C. This is the solution I used, but I actually prefer Pranay's solution better. Nevertheless, I'm showing both solutions so that you can see there is more than one way to solve such a problem. Consider **31**. If the first block is 31, the remaining cheese blocks cannot be added to 31 to make any new amounts. Similarly, for 30, 29, 28, 27, …, 17: all of these blocks will be too big (in the case of 17, the remaining blocks would need to add up to 14, but that won't cover 15 or 16). So the first block could be **16**, which leaves the remaining 4 blocks to equal 15.*

Try 1 + 1 + 1 + 12: that adds up to 15, but the 1's aren't different sized.
Try 1 + 2 + 3 + 9: this almost works, but there's no way to make 7 or 8.
*Try 1 + 2 + 4 + 8: we can make 1 (1), 2 (2), 3 (1+2), 4 (4), 5 (4+1), 6 (4+2), 7 (1+2+4), and 8 (8). Since we can make all amounts from 1-7, we can make all amounts from 8+1 through 8+7, which covers 9 through 15. Finally, since we can make all amounts from 1 through 15, we can make all amounts from 16 through 31 as well (16, 16+1, 16 +2, and all the way to 16 + 15). The cheese blocks Great Uncle Venkatesh needs are thus 1, 2, 4, 8, and 16. The product of these numbers is $1 \times 2 \times 4 \times 8 \times 16 = 64 \times 16 =$ **1024**.*

*Method 2: Logic (solution provided by Pranay Mallik, composer of this problem). The first day of the month is the 1st, so your first nugget has to be **1**. The second day is the 2nd, so you use a **2** (not 1+1 because that repeats a value). The third day is the 3rd, but 1 + 2 = 3, so you don't need a 3. You do need a **4** for the 4th day. Aha! There is a pattern here! We don't need a 5, 6, or 7, but the next one we will need comes right after 1 + 2 + 4 = 7, which is **8**. Then the next one after that will be the one that comes right after 1 + 2 + 4 + 8 = 15, which is **16**.*

*The blocks Great Uncle Venkatesh needs are 1, 2, 4, 8, and 16 (note that 1 + 2 + 4 + 8 + 16 = 31, the final day). The product of these numbers is $1 \times 2 \times 4 \times 8 \times 16 =$ **1024**.*

9) **4** **Casework/Go In Order; Average.** *This is a difficult problem unless you are very careful in the way you try different possibilities. You can focus on the number of quarters (possibly using average to get close), or you can focus on the number of pennies. Note that since the total is 100 cents (and since nickels, dimes, and quarters are always multiples of 5), the number of pennies must be a multiple of 5.*

Try 0 pennies: then 48 coins must make 100. This is not possible, because even if all the coins were as small as possible (nickels), 48 x 5 is way over 100.
Try 5 pennies: then 43 coins must make 95. 43 x 5 is again way over 95; not possible.
Try 10 pennies: then 38 coins must make 90. No.
Try 15 pennies: then 33 coins must make 85. No.
Try 20 pennies: then 28 coins must make 80. No.
Try 25 pennies: then 23 coins must make 75. No.
Try 30 pennies: then 18 coins must make 70. No, but at least that's close (18 x 5 = 90).

*Try 35 pennies: then 13 coins must make 65. **YES**. This can be accomplished with 13 nickels.*

Try 40 pennies: then 8 coins must make 60. Try all different possibilities.

- *With 2 quarters: then 6 coins must make 10. No.*
- *With 1 quarter: then 7 coins must make 35. **YES**: this can be accomplished with 7 nickels.*

- *With 0 quarters: then 8 coins must make 60.* ***YES****: this can be accomplished with 4 dimes and 4 nickels.*

<u>Try 45 pennies</u>: then 3 coins must make 55. ***YES****: this can be done with 2 quarters and 1 nickel.*

Thus, 100 cents can be made using 48 coins in ***4*** *different ways (35P+13N, 40P+1Q+7N, 40P+4D+4N, and 45P+2Q+1N).*

10) **23** **CLT, Logic.** *This is a difficult problem not because of the math, but because of the organization. Create a chart listing the digits 0 through 9 across the top (10's place) and the left side (1's place). Together, each pair of digits represents a 2-digit number (everything beginning with 0 has been shaded out since those are not two-digit numbers).*

	0	1	2	3	4	5	6	7	8	9
0		**YES**	**YES**	**YES**	**YES**	**YES**	**YES**	**YES**	**YES**	**YES**
1			**YES**						**YES**	
2		**YES**			**YES**			**YES**		
3							**YES**			
4			**YES**			**YES**			**YES**	
5					**YES**					
6				**YES**						
7			**YES**							
8		**YES**			**YES**					
9										

Go through each box to determine the sum of the digits. The number is an Awesome Number if it is divisible by the sum of its digits.

Use logic to find shortcuts when possible. Here are some:

<u>All numbers that end with 0 will definitely work</u>: 10 ÷ (1 + 0) = 10; 20 ÷ (2 + 0) = 10; etc.
<u>All 2-digit prime numbers definitely will not work</u> and can be eliminated straight away.
<u>All multiples of 9 will work</u> due to the divisibility rule for 9.

*We conclude that there are **23** two-digit Awesome Numbers.*

11) **40** **Cryptarithms, Magic Squares: Logic.** *A great problem by Nathaniel, which we will call a "cryptamagic" problem because it involves both a cryptarithm as well as a Magic Square.*

We know the grid is a Magic Square because it tells us that the sum of each row, column, and diagonal is the same.

<u>Begin with the cryptarithm, and start with the most restrictive statement</u>. We know that O = 0, so fill that in right away. Also, G must be 1 because when two 4-digit numbers add up to a 5-digit number, the leading digit can only be 1.

The most restrictive statement we have is that both C and E are neither prime nor square. This means that C and E are not 2, 3, 5, 7 or 0, 1, 4, 9. The only digits left are 6 and 8. Try C=6 and E=8. Then F must be 4 (since 6 + 4 = 10, which ends with 0). We get:

```
   A B 6 6
+  D 8 4 4
 1 0 0 1 0
```

But now we have a problem: in the 100's place, there is a carryover, so 1 + B + 8 must end with 0, which means that B + 9 = 10. But then B=1, and that is not possible since G already is 1. This means that C=6 and E=8 does not work, so C=8 and E=6 MUST work.

Then F must be 2, and B must be 3. We get:

```
   A 3 8 8
+  D 6 2 2
 1 0 0 1 0
```

In the 1000's place, there is a carryover, so A + D + 1 = 10. Then A + D = 9. We have used the digits 0, 1, 2, 3, 6, and 8, leaving us with only 4, 5, 7, and 9. The only pair that adds up to 9 is 4 + 5. In the cryptarithm, A could be 4 and D could be 5, or vice versa. But consider the Magic Square: A + B + C forms a line. If A is 5, then 5 + 3 + 8 = 16, which is impossible for a line of a Magic Square because all lines must add up to a multiple of 3 (one of the rules for Magic Squares is that the sum of any row, column, or diagonal is equal to 3 times the middle number). This means that A must be 4 and D must be 5. The completed cryptarithm is:

```
   4 3 8 8
+  5 6 2 2
 1 0 0 1 0
```

<u>In the Magic Square</u>, the middle number is D, which is 5. This means that the sum of each line must be 3 x 5 = 15. Since G=1, the other digit in the middle row must be 9 (because 9 + 5 + 1 = 15). Now focus on H. H cannot be any of the digits used in the cryptarithm because

H was not part of the cryptarithm. The only digits not used in the cryptarithm were 7 and 9. Since 9 was already used in the Magic Square, H must be 7, and the number directly below D (in the same column as H) must be 3 (because 7 + 5 + 3 = 15). So far, we have

	7	
9	5	1
	3	

The bottom row must be A, B, and C, because B is already listed (3). We know that A=4 and C=8. The bottom left can't be 8 because then 9 + 8 > 15 for the left column. So the bottom left must be 4, the bottom right must be 8, and as a result, the top left must be 2 and the top right must be 6. The completed Magic Square is

2	7	6
9	5	1
4	3	8

*Indeed, all rows, columns, and diagonals add up to 15. The problem asks for the product of the 2nd and 3rd digits in the top row minus the 1st digit in the top row. That's (7 x 6) – 2 = **40**.*

12) **Deep Dish Crust with Ham** **CLT, Logic.** *We like to call this type of pure logic question a "Nelson Special." Kudos to Kavya Kaushal for creating this year's excellent version!*

Begin by creating a CLT of all options. For this chart, toppings are on top, and crusts are on the side. Options that are not available have been shaded.

	Veggies	Pepperoni	Cheese	Ham	Bacon	Pineapple
Thin						
Thick						
Deep Dish						
Gluten Free						

Start with José. Since each crust has more than one type of topping, José doesn't know which pizza is Matty's favorite. However, when he states that he knows that Ricardo also does not know Matty's favorite pizza, he reveals some useful information. If José knew the pizza was thin crust or thick crust, he could not say for certain that Ricardo didn't know which pizza was Matty's favorite, because the thin crust with bacon is the only pizza with bacon, while the thick crust with pepperoni is the only pizza with pepperoni. In those two cases, Ricardo would indeed know Matty's favorite pizza because Ricardo knows the topping, and if the topping were bacon or pepperoni, Ricardo would know Matty's favorite. Since José is certain that Ricardo does not know, we know for sure that Matty's favorite pizza can't be thin crust or thick crust. Eliminate all pizzas in those categories and update the chart:

	Veggies	Pepperoni	Cheese	Ham	Bacon	Pineapple
Thin						
Thick						
Deep Dish						
Gluten Free						

Now focus on Ricardo. Ricardo originally didn't know which pizza Matty liked best, but now he does. This means that the pizza must correspond to a topping that only appears one time in the chart above: veggies, ham, or pineapple. It cannot be cheese because then Ricardo couldn't say he knew which pizza Matty liked. Update the chart once again:

	Veggies	Pepperoni	Cheese	Ham	Bacon	Pineapple
Thin						
Thick						
Deep Dish						
Gluten Free						

Use the final statement by José. After Ricardo's statement, José also knows which pizza is Matty's favorite. This means that the crust cannot be gluten free, because there are two types of gluten free pizzas still left: if José knew that the crust was gluten free, he still wouldn't know Matty's favorite. But he does know Matty's favorite, so the crust has to be deep dish.

*Matty's favorite pizza is therefore **Deep Dish Crust with Ham**.*

Question Composition Credits

Problem	Composer
1	Tejas Rao
2, 5-7	Mr. G.
3	Chiran Arumugam
4	Sneha Kancharla
8	Pranay Mallik
9	Ankith Madadi
10	Dylan Frake
11	Nathaniel Satriya (TA)
12	Kavya Kaushal (TA)