1) **100** **Grouping; Distributive Property.** *This is one of those problems when it pays to LOOK at the numbers before you start calculating. Do you notice something about the numbers being added and the numbers being subtracted? It's the same set of numbers! We have:*

(5 × 18) – (4 × 18). Using Distributive Property, that's the same as (5 – 4) × 18, which is just 1 × 18, or 18. Do the same thing for the 19's, the 20's, the 21's, and the 22's. You get 18 + 19 + 20 + 21 + 22, which is ***100****.*

2) **133** **Work Backwards.** *This is a classic WB (Work Backwards) problem: we are given the final result (109) but need to find the original number. To do this, you perform the opposite operation and go one step at a time.*

Add 10: Subtract 10. 109 – 10 = 99.
Multiply by 9: Divide by 9. 99 / 9 = 11.
Subtract 8: Add 8. 11 + 8 = 19.
Divide by 7: Multiply by 7. 19 x 7 = ***133****.*

Confirm by checking. 133 / 7 = 19 – 8 = 11 x 9 = 99 + 10 = 109: YES.

3) **139** **CLT; Logic; RTQ!** *Probably the most difficult part of this problem is making sure that you read it correctly. We want the sum of the digits of two consecutive numbers to be an even number greater than 20. Begin with 50 and 51 and keep going until you notice something.*

50 & 51: 5 + 6 = 11: odd. 51 & 52: 6 + 7 = 13: odd. 52 & 53: 7 + 8 = 15: odd.
53 & 54: 8 + 9 = 17: odd. 54 & 55: 9 + 10 = 19: odd. 55 & 56: 10 + 11 = 21: odd.

So far, all we are getting are odd results. Let's consider: is there a reason for this? In each of the above cases, we are adding exactly three odd digits and one even digit. Aha! Odd + odd + odd + even = odd, so in the 50's, our sum will always be odd. Move to 59 & 60.

59 & 60: 14 + 6 = 20. That's even, but it's not greater than 20. If we do 60 & 61, we now have even + even + even + odd, which is always going to be odd. SKIP to the next pair of numbers similar to 59 & 60. 69 & 70: 15 + 7 = 22. YES. The two numbers we need are 69 and 70. The questions asks for their sum: 69 + 70 = ***139****.*

4) **3** **LCM.** *Rephrase the question so it's easier: groups of 4 with remainder 3 means that the total number of forks is divisible by 4 with R3. The same group of forks is also divisible by 6 with R3. Any number that is divisible by 4 AND 6 is divisible by the LCM of 4 and 6. The LCM of 4 and 6 is 12, so it follows that the number of forks, when put in groups of 12, will always have a remainder of* ***3****.*

Another way this can be solved is to figure out a possible number of forks. Use two CLT's until you see a number that works for both lists.

Div. by 4, R3: 7, 11, 15, 19, 23, 27, etc. Div. by 6, R3: 9, 15, etc. The first common number is 15. If you have 15 forks and put them in groups of 12, you get 1 group with 3 left over.

5) **6** **Logic; CLT or G/C.** *The first thing to do on questions like these is to start with the largest denomination, because that limits how many tries you have to make. In this case, quarters*

are the largest.

- *MohNay can't have 4 quarters because that's over 87 cents.*
- *If she had 3 quarters, then she would have 6 dimes. But then 75 + 60 is way over 87.*
- *If she had 2 quarters, then she would have 4 dimes. But then 50 + 40 = 90, which is once again over 87.*

It follows that MohNay MUST have either 0 or 1 quarters. 0 is not likely, because then she would also have 0 dimes, and the question would be rather silly. So let's try 1 quarter and see if that works.

1 Q means 2 D. 25 + 20 = 45 cents so far. 87 – 45 = 42 cents left. MohNay needs twice as many pennies as nickels. Try 5 N and 10 P: 25 + 10 = 35. That's close, but a bit short. Try 6 N and 12 P: 30 + 12 = 42. That's it! MohNay has ***6*** *nickels.*

6) **160** **Area of Squares; Perimeter.** *Begin by COUNTING the number of squares: there are 28. Next, divide 700 by 28 to get the area of each square. 700 / 28 = 25. With an area of 25 sq meters, the side of each square must be 5, because 5 x 5 = 25.*

Now, COUNT the total number of sides along the H-shrine. Count carefully, because there are a lot of sides!

There are 32 total sides, each of length 5 meters. Finally, multiply: 32 x 5 = ***160****, which is the perimeter of the H-shrine.*

7) **13** **Simultaneous Equations.** *WRITE OUT THE BULLET POINTS! It makes this problem much, much easier to understand.*

- *2W + 3A = 31*
- *3T + 4A = 27*
- *4T + 5W = 75*
- *Sude needs 2W + 2A + 2T*
- *She has $25*

Just looking at the bullet points suggests the right strategy: Simultaneous Equations. Combine all three equations to see what you get, and go from there.

$$\begin{array}{r} 2W + 3A = 31 \\ 3T + 4A = 27 \\ \underline{+ \quad 4T + 5W = 75} \\ 7T + 7A + 7W = 133 \end{array}$$

Look at the sum: 7T + 7A + 7W. Does that look like what Sude needs? Yes it does! She needs 2 of each fruit; we know the price for 7 of each fruit. Divide by 7 first:

$$\frac{7T + 7A + 7W}{7} = \frac{133}{7}$$

$$1T + 1A + 1W = 19$$

The price for one of each fruit is $19, so the price for two of each fruit is two times that.

19 x 2 = 38, which is the price for what Sude needs.

*BE CAREFUL: that's not what the question asks! Sude has $25. The question is asking how much MORE money she will need to pay for the fruit. Subtract: $38 – $25 = $**13**.*

8) **119** **RTQ.** *At first this seems like a strange question, but if you read it carefully, you will realize that it's nothing more than an addition problem! Be sure you understand what to do by reading the example, then apply that understanding to the problem itself.*

<u>Starting value of T</u>: 2.
<u>T to H</u>: 12. <u>H to E</u>: 3.
<u>E to T</u>: 15. <u>T to I</u>: 11. <u>I to N</u>: 5. <u>N to G</u>: 7.
<u>G to G</u>: 0. <u>G to O</u>: 8. <u>O to E</u>: 10. <u>E to S</u>: 14.
<u>S to S</u>: 0. <u>S to K</u>: 8. <u>K to R</u>: 7. <u>R to A</u>: 17. <u>A to A (all remaining combinations)</u>: 0.

*The total value is 2 + 12 + 3 + 15 + 11 + 5 + 7 + 0 + 8 + 10 + 14 + 0 + 8 + 7 + 17 + 0. Whenever you have a long list of numbers to add, be sure to add it in TWO DIFFERENT WAYS to make sure you get the same answer both times. Often, the best way to do this step is to group by 10's, 20's, 30's, etc. We get 10 + 10 + 20 + 30 + 25 + 24 = **119**.*

9) **65292** **Logic (Secret Password), G/C.** *As with all Secret Password questions, begin by setting up the number of spaces (in this case five, since it has five digits) along with your Box O' Digits.*

<u>Begin with the most restrictive statement</u>. The first digit is three times the 5th digit. This means that the 1st digit can be 0, 3, 6, or 9; eliminate 0 because then the password isn't a 5-digit number. The corresponding 5th digit would be 1, 2, or 3.

<u>Use the next most restrictive statement</u>. The fourth digit is 4 more than the 2nd digit. This means that the 4th digit can be anything from 4-9, while the corresponding 2nd digit ranges from 0-5.

<u>Combine the remaining statement with what we know from the statement above</u>. The 3rd digit is 3 less than the 2nd digit. This means that the 3rd digit can be anything from 0 to 6, while the corresponding 2nd digit would be anything from 3 to 9. But we know from the second statement that the 2nd digit can only be from 0-5. The second digit must work with both statements, so it must be 3, 4, or 5. This means that the 4th digit must be 7, 8, or 9.

Summarize the information into your spaces:

___	___	___	___	___
3, 6, 9	*3, 4, 5*	*0, 1, 2*	*7, 8, 9*	*1, 2, 3*

At first, this might seem like a huge number of possibilities, but keep in mind that the 1st and 5th digits must go together, while the 2nd, 3rd, and 4th digits must go together as well. There are 3 possible groupings for the 1st and 5th digits, and there are 3 possible groupings for the 2nd, 3rd, and 4th digits. That gives us 3 x 3 = 9 possible answers. List these:

33071
34181
35291

63072
64182
65292
93073
94183
95293

*Of these 9 possibilities, only one includes three different pairs of digits that add up to 11. 6 + 5 = 11; 2 + 9 = 11; 9 + 2 = 11. The code to Sude's safe is **65292**.*

10) **40** **Logic; Process of Annihilation.** *Originally, I solved this problem by focusing on the middle and right columns, listing all possible digits that could go in those boxes. I determined that the only two digits that didn't work for any of those boxes were 5 and 7; therefore, those digits had to go in the left column. I then figured out which numbers went in which box from there.*

*That method works, and it did lead to the correct answer. However, there is a more direct method which is faster and easier. **Nathaniel** provides it as follows:*

This problem has a lot of different restrictions. To make it easier from the start, <u>let's begin solving by using the most restrictive fact: that $C \times V \div Z = 27$</u>. We can see from this that $C \times V$ has to be a multiple of 27, so C and V have to be either 9 and 3 or 9 and 6.

*If they are 9 and 3, Z must be 1. However, since $X \div Y - Z = 0$, there is a problem: if Z is 1, then $X \div Y$ must also be 1, which means that $X = Y$. That is not possible, so **Z must be 2**, and C and V are 9 and 6.*

*Now since $T \div U - V = 1$, V must be 6: if it were 9, then $T \div U = 10$, which is again impossible. Therefore, **$V = 6$, meaning that $C = 9$** and $T \div U = 7$. It is then clear that **T is 7 and U is 1**, as those are the only digits whose quotient is 7.*

*Now, notice that $X \div Y - Z = 0$. Since $Z = 2$, $X \div Y$ must also be 2. Since 2 and 6 have already been used, **X and Y have to be 8 and 4 respectively**.*

*Finally, because $B \div U \times Y = 12$, **B is 3** ($3 \times 4 = 12$) **and A, the last remaining digit, is 5** ($5 + 3 + 9 = 17$). Our final answer, $Y \times A \times Z$, is $4 \times 5 \times 2$, or <u>**40**</u>.*

The completed chart looks like this:

5	**3**	**9**
7	**1**	**6**
8	**4**	**2**

11) **23** **CLT, Logic.** *This is a difficult problem not because of the math, but because of the organization. Create a chart listing the digits 0 through 9 across the top (10's place) and the left side (1's place). Together, each pair of digits represents a 2-digit number (everything beginning*

with 0 has been shaded out since those are not two-digit numbers).

Go through each box to determine the sum of the digits. The number is an Awesome Number if it is divisible by the sum of its digits.

Use logic to find shortcuts when possible. Here are some:

<u>All numbers that end with 0 will definitely work</u>: 10 ÷ (1 + 0) = 10; 20 ÷ (2 + 0) = 10; etc.
<u>All 2-digit prime numbers definitely will not work</u> and can be eliminated straight away.
<u>All multiples of 9 will work</u> due to the divisibility rule for 9.

	0	1	2	3	4	5	6	7	8	9
0		**YES**	**YES**	**YES**	**YES**	**YES**	**YES**	**YES**	**YES**	**YES**
1			**YES**						**YES**	
2		**YES**			**YES**			**YES**		
3							**YES**			
4			**YES**			**YES**			**YES**	
5					**YES**					
6				**YES**						
7			**YES**							
8		**YES**			**YES**					
9										

*There are **23** two-digit Awesome Numbers.*

12) **Deep Dish Crust with Ham** **CLT, Logic.** *We like to call this type of pure logic question a "Nelson Special." Kudos to Kavya Kaushal for creating this year's excellent version!*

Begin by creating a CLT of all options. For this chart, toppings are on top, and crusts are on the side. Options that are not available have been shaded.

<table>
<tr><th></th><th>Veggies</th><th>Pepperoni</th><th>Cheese</th><th>Ham</th><th>Bacon</th><th>Pineapple</th></tr>
<tr><th>Thin</th><td></td><td bgcolor="#a6a6a6"></td><td></td><td></td><td></td><td></td></tr>
<tr><th>Thick</th><td bgcolor="#a6a6a6"></td><td></td><td bgcolor="#a6a6a6"></td><td></td><td bgcolor="#a6a6a6"></td><td></td></tr>
<tr><th>Deep Dish</th><td bgcolor="#a6a6a6"></td><td bgcolor="#a6a6a6"></td><td></td><td></td><td bgcolor="#a6a6a6"></td><td bgcolor="#a6a6a6"></td></tr>
<tr><th>Gluten Free</th><td></td><td bgcolor="#a6a6a6"></td><td></td><td bgcolor="#a6a6a6"></td><td bgcolor="#a6a6a6"></td><td></td></tr>
</table>

<u>Start with José</u>. Since each crust has more than one type of topping, José doesn't know which pizza is Matty's favorite. However, when he states that he knows that Ricardo also does not know Matty's favorite pizza, he reveals some useful information. If José knew the pizza was thin crust or thick crust, he could <u>not</u> say for certain that Ricardo didn't know which pizza was Matty's favorite, because the thin crust with bacon is the only pizza with bacon, while the thick crust with pepperoni is the only pizza with pepperoni. In those two cases, Ricardo would indeed know Matty's favorite pizza because Ricardo knows the topping, and if the topping were bacon or pepperoni, Ricardo would know Matty's favorite. Since José is certain that Ricardo does not know, we know for sure that Matty's favorite pizza can't be thin crust or thick crust. Eliminate all pizzas in those categories and update the chart:

<table>
<tr><th></th><th>Veggies</th><th>Pepperoni</th><th>Cheese</th><th>Ham</th><th>Bacon</th><th>Pineapple</th></tr>
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</table>

<u>Now focus on Ricardo</u>. Ricardo originally didn't know which pizza Matty liked best, but now he does. This means that the pizza must correspond to a topping that only appears one time in the chart above: veggies, ham, or pineapple. It cannot be cheese because then Ricardo couldn't say he knew which pizza Matty liked. Update the chart once again:

<table>
<tr><th></th><th>Veggies</th><th>Pepperoni</th><th>Cheese</th><th>Ham</th><th>Bacon</th><th>Pineapple</th></tr>
<tr><th>Thin</th><td bgcolor="#a6a6a6"></td><td bgcolor="#a6a6a6"></td><td bgcolor="#a6a6a6"></td><td bgcolor="#a6a6a6"></td><td bgcolor="#a6a6a6"></td><td bgcolor="#a6a6a6"></td></tr>
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</table>

<u>Use the final statement by José</u>. After Ricardo's statement, José also knows which pizza is Matty's favorite. This means that the crust cannot be gluten free, because there are two types of gluten free pizzas still left: if José knew that the crust was gluten free, he still wouldn't know Matty's favorite. But he does know Matty's favorite, so the crust has to be deep dish.

Matty's favorite pizza is therefore ***Deep Dish Crust with Ham***.

Question Composition Credits

Problem	Composer
1, 3–7, 9–10	Mr. G.
2	Tejas Rao
8	Chiran Arumugam
11	Dylan Frake
12	Kavya Kaushal (TA)